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**Mathematics**  
**Higher level**  
**Paper 1**

Monday 13 May 2019 (afternoon)

Candidate session number

2 hours

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**Instructions to candidates**

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**.





2. [Maximum mark: 4]

Determine the first three terms of  $(1 - 2x)^{11}$  in ascending powers of  $x$ , giving each term in its simplest form.

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16EP03

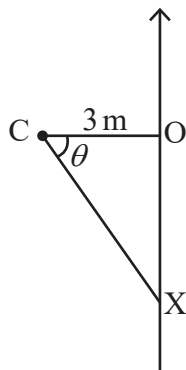
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5. [Maximum mark: 6]

A camera at point  $C$  is 3 m from the edge of a straight section of road as shown in the following diagram. The camera detects a car travelling along the road at  $t = 0$ . It then rotates, always pointing at the car, until the car passes  $O$ , the point on the edge of the road closest to the camera.



A car travels along the road at a speed of  $24 \text{ ms}^{-1}$ . Let the position of the car be  $X$  and let  $\widehat{OCX} = \theta$ .

Find  $\frac{d\theta}{dt}$ , the rate of rotation of the camera, in radians per second, at the instant the car passes the point  $O$ .

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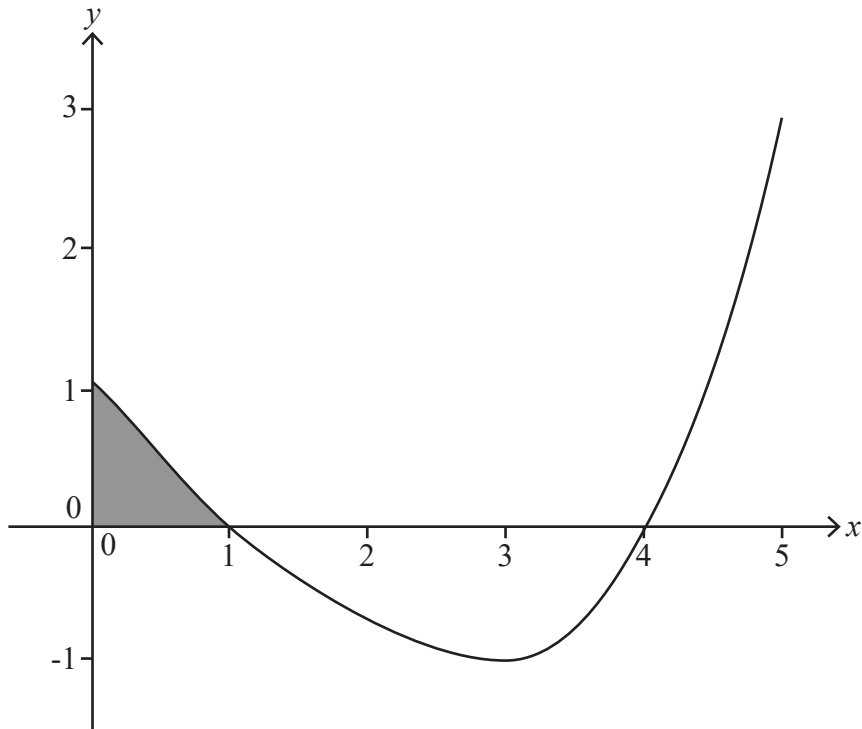


16EP08



8. [Maximum mark: 9]

The graph of  $y = f'(x)$ ,  $0 \leq x \leq 5$  is shown in the following diagram. The curve intercepts the  $x$ -axis at  $(1, 0)$  and  $(4, 0)$  and has a local minimum at  $(3, -1)$ .



(a) Write down the  $x$ -coordinate of the point of inflexion on the graph of  $y = f'(x)$ . [1]

The shaded area enclosed by the curve  $y = f'(x)$ , the  $x$ -axis and the  $y$ -axis is  $0.5$ .  
Given that  $f(0) = 3$ ,

(b) find the value of  $f(1)$ . [3]

The area enclosed by the curve  $y = f'(x)$  and the  $x$ -axis between  $x = 1$  and  $x = 4$  is  $2.5$ .

(c) Find the value of  $f(4)$ . [2]

(d) Sketch the curve  $y = f(x)$ ,  $0 \leq x \leq 5$  indicating clearly the coordinates of the maximum and minimum points and any intercepts with the coordinate axes. [3]

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16EP10



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### Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 15]

(a) Show that  $(\sin x + \cos x)^2 = 1 + \sin 2x$ . [2]

(b) Show that  $\sec 2x + \tan 2x = \frac{\cos x + \sin x}{\cos x - \sin x}$ . [4]

(c) Hence or otherwise find  $\int_0^{\frac{\pi}{6}} (\sec 2x + \tan 2x) dx$  in the form  $\ln(a + \sqrt{b})$  where  $a, b \in \mathbb{Z}$ . [9]

10. [Maximum mark: 15]

The function  $p(x)$  is defined by  $p(x) = x^3 - 3x^2 + 8x - 24$  where  $x \in \mathbb{R}$ .

(a) Find the remainder when  $p(x)$  is divided by

(i)  $(x - 2)$

(ii)  $(x - 3)$ . [3]

(b) Prove that  $p(x)$  has only one real zero. [4]

(c) Write down the transformation that will transform the graph of  $y = p(x)$  onto the graph of  $y = 8x^3 - 12x^2 + 16x - 24$ . [2]

The random variable  $X$  follows a Poisson distribution with a mean of  $\lambda$  and  $6P(X = 3) = 3P(X = 2) - 2P(X = 1) + 3P(X = 0)$ .

(d) Find the value of  $\lambda$ . [6]



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11. [Maximum mark: 20]

Two distinct lines,  $l_1$  and  $l_2$ , intersect at a point P. In addition to P, four distinct points are marked out on  $l_1$  and three distinct points on  $l_2$ . A mathematician decides to join some of these eight points to form polygons.

- (a) (i) Find how many sets of four points can be selected which can form the vertices of a quadrilateral.
- (ii) Find how many sets of three points can be selected which can form the vertices of a triangle.

[6]

The line  $l_1$  has vector equation  $\mathbf{r}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \lambda \in \mathbb{R}$  and the line  $l_2$  has vector equation

$$\mathbf{r}_2 = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 6 \\ 2 \end{pmatrix}, \mu \in \mathbb{R}.$$

The point P has coordinates (4, 6, 4).

- (b) Verify that P is the point of intersection of the two lines.

[3]

The point A has coordinates (3, 4, 3) and lies on  $l_1$ .

- (c) Write down the value of  $\lambda$  corresponding to the point A.

[1]

The point B has coordinates (-1, 0, 2) and lies on  $l_2$ .

- (d) Write down  $\vec{PA}$  and  $\vec{PB}$ .

[2]

Let C be the point on  $l_1$  with coordinates (1, 0, 1) and D be the point on  $l_2$  with parameter  $\mu = -2$ .

- (e) Find the area of the quadrilateral CDBA.

[8]



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16EP14

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16EP15



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